CONVEXITY

It is important to completely understand the price movement of fixed income securities in order to take full advantage of their potential return. It has always been understood that a bond's price changes with its yield. However, the rate or amount that a bond's price fluctuates with changes in yield depends upon the cash flow characteristics of the security and the magnitude of the movement in interest rates. Until the concept of convexity was developed, it had been a mystery why some bonds gained value more slowly when rates declined and lost value more quickly when rates rose.

Duration is a concept that explains a portion of a bond's price movement. The effective duration of a bond tells us how much the price will change with small changes in interest rates. Duration is a measure of the life of a bond, an average of the time intervals to each cash flow payment (interest and principal) weighted by the discounted present value of each payment. Duration does not always supply the full picture for larger changes in interest rates (over 50 basis points). This is particularly true for bonds with embedded options.

The concept of convexity, when combined with duration, helps to explain the price movement of a bond during large shifts in interest rates, assuming a parallel shift in the yield curve.* Convexity is a measure of the rate of change in duration, and thus, it measures how much a bond's price/yield relationship deviates from a straight line. Put another way, convexity exists when the relationship between a bond's price and interest rate movement

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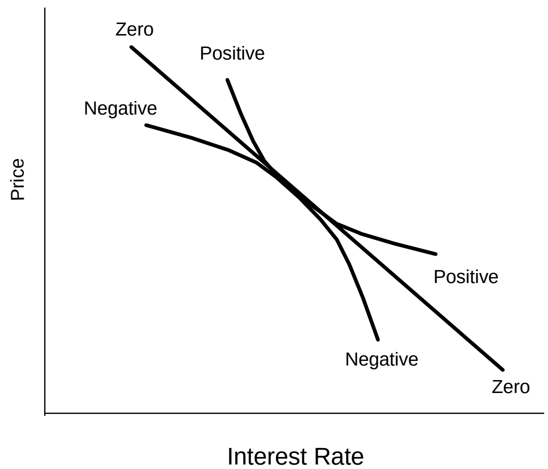
* Mathematically, the relationship between price changes, duration, and convexity is expressed as follows:

$$\% \triangle p = - \bar{D} \cdot \triangle y + 1/2 \cdot c \cdot \triangle y^2$$

Where p = price
D = duration
y = yield
c = convexity

is not linear (Exhibit I.). For example, a bond has positive (negative) convexity if its price increase for a 100 basis point (bp) decline in interest rates is greater (less) than its price decrease for a similar 100 bp rise in rates.

Convexity **(EXHIBIT I.)**



The following are three general statements that can be made about the convexity of bonds which are free from options, sinking funds, and credit concerns:

1. For equal duration securities, higher coupon bonds have better convexity than lower and zero coupon issues.

2. For equal maturity issues, zeros have the most convexity.

3. For equal duration portfolios, a barbell structure has better convexity than a bullet structure.

The first statement is obvious. To maintain the same duration as a zero coupon security, it is necessary to lengthen maturity as the coupon increases. This lengthens the final principal payment and increases the dispersion of the security's cash flow. In time, this increases the volatility characteristics of the issue and its convexity. Put another way, a coupon bond can be viewed as a series of zeros with varying maturities. As rates rise, the higher the coupon bond (for equal duration) will be higher and the reduction in duration will be greater. A zero coupon bond's duration will remain virtually unchanged. In this environment, the price of the coupon bond will decrease less than that of the zero bond, reflecting its more positive convex characteristic.

The second statement is accurate since the final bond principal payment (at maturity) has the most convexity and the cash flow of a zero is entirely composed of its final payment. Therefore, it follows that a zero will have greater convexity than a coupon-bearing bond with the same maturity. In other words, even though the duration of these two securities differs, a coupon bond having a maturity equal to a zero bond will have less convexity because a smaller portion of the bond's value is represented by the final and most convex payment.

The validity of statement three follows directly from statement one. For example, a three-year Treasury has low effective convexity, but, in combination with an investment in a thirty-year Treasury bond has higher convexity than a portfolio comprised solely of a 10-year bullet Treasury. Put another way, the duration-convexity relationship is not linear. For a given increase in the level of duration, there will be a greater increase in convexity. The percentage difference in duration between the 10-year Treasury note and the 30-year Treasury bond is 64%; however, convexity is increased by 253%. Lengthening a security's maturity, and therefore its final principal payment, substantially increases convexity.

Negative Convexity

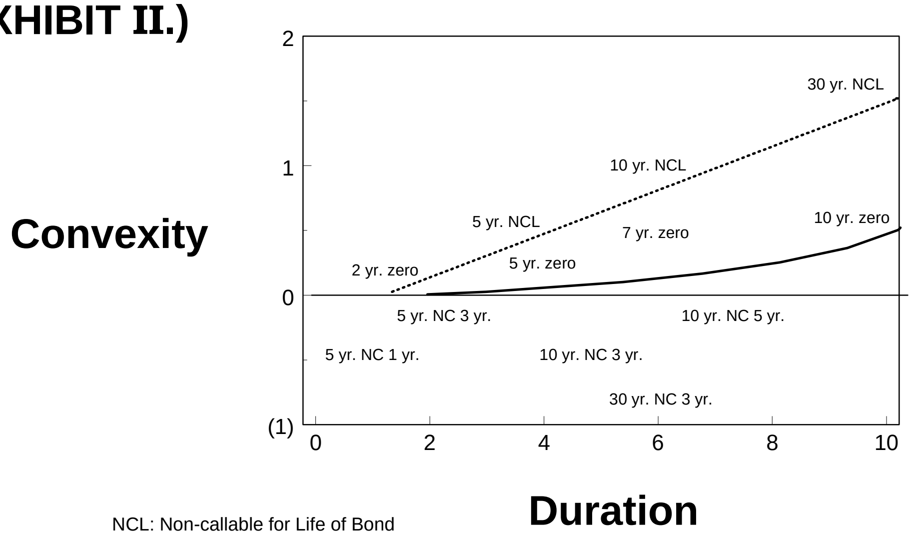
A negatively convex portfolio will provide an investor with a higher initial yield than that of a positively convex portfolio. The offset to that higher yield, however, is that the negatively convex portfolio will experience unfavorable changes in duration as yields change. In other words, by creating a negatively convex portfolio, the investor is gaining higher initial yield at the cost of potentially poor price performance. The following points discuss the implications of negative convexity in an environment of stable, falling and rising interest rates:

- **Stable Interest Rates.** The tradeoff between higher yield and negative convexity makes sense if the investor is confident of interest rate stability (i.e. interest rates remain in a trading range.) If rates remain unchanged or experience little movement, the higher yield inherent in the negatively convex security will compensate the portfolio for the lack of price movement. Secondly, the portfolio would not be adversely affected if a bond were to mature or be called, since the reinvestment rate would be roughly the same. In addition, the stability of interest rates would preclude negative price returns from overwhelming the initial yield advantage. In this case, negative convexity enhances the returns of the portfolio.
- **Falling Interest Rates.** In a falling rate scenario, the investor would seek to avoid negative convexity. When rates are falling in a **significant fashion**, the potential for bonds to be called is heightened. Since rates are lower, the reinvestment rate may not be able to offset the initial yield advantage. Secondly, since the bond is negatively convex, its price performance will lag that of a comparable duration, less negatively (or positively) convex bond.
- **Rising Interest Rates.** When rates **increase by a significant amount**, bond issuers lose incentive to call bonds early. This would effectively terminate cheap, long-term financing. This causes the duration of a bond to extend past original expectations. In this case, the investor is faced with a "Hobson's Choice" of holding the bond and foregoing available higher yields, or selling the bond at a loss in order to purchase a new, higher yielding asset. If he chooses to sell, the pain is exacerbated because the

negative convexity of the bond would have caused it to underperform its duration neutral, less convex counterpart.

(Exhibit II.) illustrates the performance of bonds that have different characteristics. For example, non-callable, long maturity and long duration bonds have large positive convexities. Callable bonds with little call protection have large negative convexities.

(EXHIBIT II.)



At , we closely analyze the convexity properties of securities. We strive, ceteris paribus, to capture the relative value of convexity in our portfolios**. . . .**

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